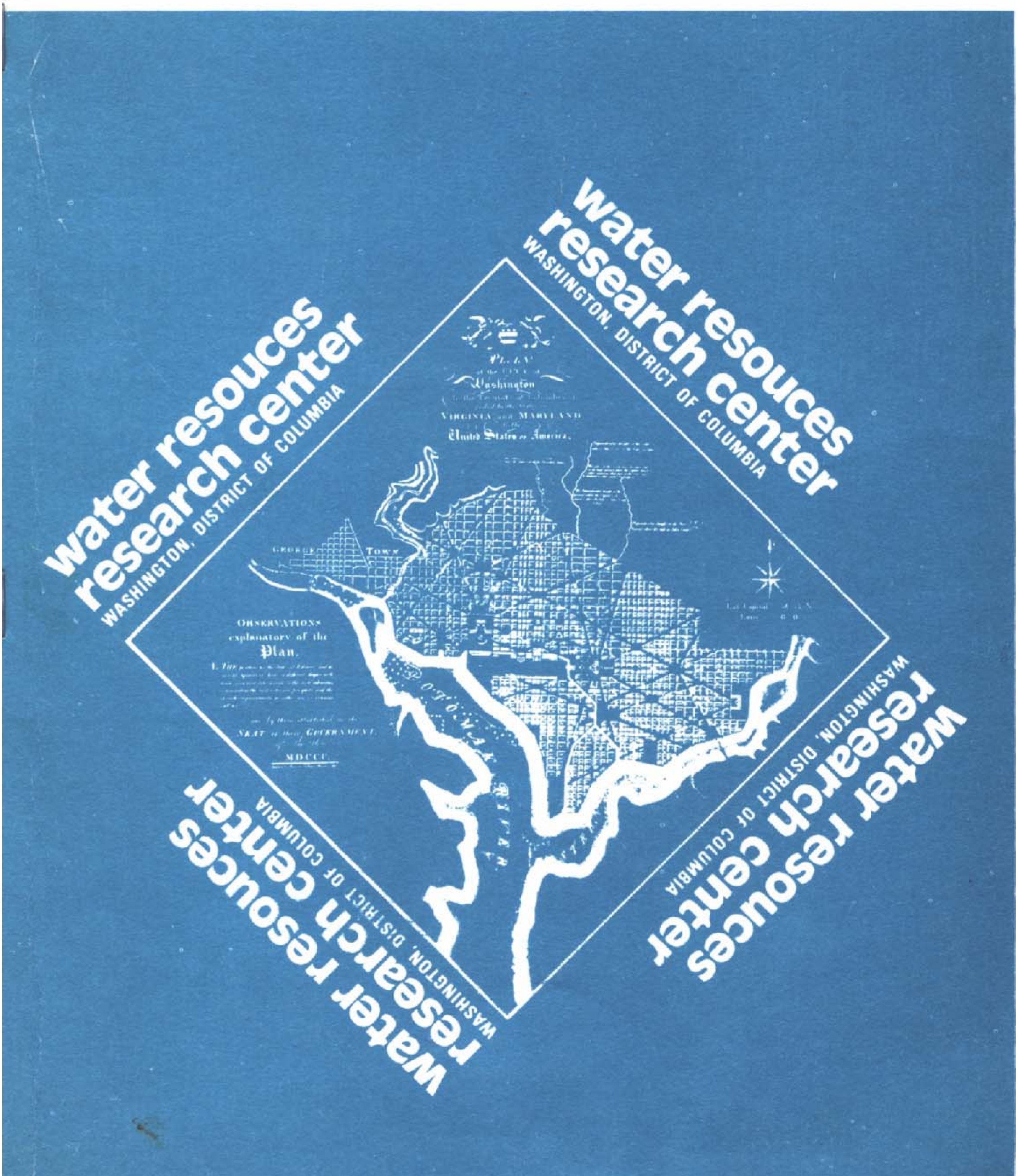
PERSISTENCE MODELING FOR THE POTOMAC AND ADJACENT RIVERS

G. K. Young
R. U. Jettmar

WRRC Report No. 12

UNIVERSITY OF THE DISTRICT OF COLUMBIA



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By

G.K. Young [1] and R.U. Jettmar[2]

FINAL REPORT

Project B-008-DC

The work upon which this publication is based was supported by the Office of Water Research and Technology, U.S.; Department of the Interior, under the provisions of Public Law 88-379, as amended.

Agreement Number 14-34-0001-6067

Water Resources Research Center
Washington Technical Institute
Washington, DC 20008

September 1977

1. Principal Investigator, 2. Research Associate

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Objective

The objective of this research is to analyze a new strategy, for generation of synthetic stream flows. The new strategy involves the fitting of theoretic autocorrelation functions to empiric functions by least squares[17]. The supposition is that such a procedure will improve Monte Carlo models in the ability to produce drought characteristics with synthetic data that approximate reality better than other available alternatives.

The analysis is conducted on a case study basis on data pertaining to:

1. The Patuxent River near Unity, MD (34.8 sq. mi. drainage).

2. The Potomac River at Point of Rocks, MD (9651 sq. mi. of drainage).

3.. The Rappahannock River near Fredericksburg, VA (1599 sq. mi. of drainage).

All three sites are predominantly unregulated and it is assumed that the historic data are stationary in a statistical context.

Previous research at Catholic University laid the groundwork for this research[8]. The use of stream flow generating models is a practical way to determine probabilities and confidence intervals for drought assessment. Such models are strongly dependent upon the proper consideration of hydrologic persistence. The previous research identified another possibly better approach for persistence modeling. The purpose of this current research is to analyze the improve approach and test is using the case study stream flow data. The new method appears to provide an advancement of the state of-the-art of persistence modeling. Two products of the research are generated. One, an evaluation of an improved estimation method for estimation of regional drought probabilities. And two, a basis from which to develop further generalizations in order to launch a national study to extend the results to the entire United States.

In short, the orientation of this research is on fitting persistence models with least squares matching of theoretic autocorrelation functions to empiric autocorrelations. Auto correlations greater than lag one, and, in the case of self similar models, much greater than one, contain information and "function fitting" should capture this information. Fitting theoretic covariance functions should impart more persistence to statistical models and produce synthetic droughts of greater severity, more in accordance with empiric evidence. This research is oriented toward testing this premise. Statistical preservation of critical periods in synthetic data is the goal. The change from existing practice is a shift in methods from using point estimates of persistence parameters to function fitting of theoretic covariance functions. This research produces sufficient empiric evidence to indicate that a. shift in emphasis may accomplish the goal of improved persistence modeling.

Background

The drought estimation problem is of national significance and could influence basis approaches to the hydrologic studies supporting water resources planning. Reservoir de sign is and will continue to be oriented to the conservation of water resources growing scarcer by the year. Drought design becomes a very significant problem <u>in times of drought;</u> in times of abundance interest wanes.

Relatively few researches are directly working in this area 13 and it is very specialized. Complete and up to date references are available[10, 16]. Most other works are involved in large scale Monte Carlo experiments to determine sampling statistics for known population models. The work herein is directed at the inverse problem; given a limited *sample,* how does one fit persistence models that agree to the extent possible with measured data with respect to drought attributes. Why is this specialized technical area of such interest?

Because, simply put, critical period or drought analysis , of stream flow *and* synthetic data generation are not compatible. This statement applies to the current state-of-the-art. This research further studies methods to achieve compatibility which were identified, and partially tested, in a recent,' previous OWRT-Title II research project, "Hydrologic Estimation Procedures in Water Resources Planning," conducted
at Catholic University. To understand the incompatibility, it is necessary to discuss critical period analysis, the role of synthetic data generation, and a proposed new parameter fitting method for stream flow persistence modeling which holds significant promise for advancing the state-of-the-art for practicing water resource planners.

What is critical period analysis? It *is* a practical engineering *approach* to the problem of reservoir design to meet water requirements. The requirements may be for water supply, irrigation power generation, etc. *Most* engineers follow these steps:;

1. The requirements are *specified:*

2. The historic stream flow records, possible including evaporation data are gathered and adjusted to the reservoir site.

3. A routing study is undertaken that routes the adjusted data through the reservoir deducting evaporation, saving excesses, meeting requirements and identifying the minimum storage necessary: To facilitate the analysis of alternative requirements and reservoir sizes, that portion of the record that produces maximum drawdown from a full condition till the time of

maximum storage depletion, is identified as the "critical period." The non-critical remainder of the historic record is not used in routing analysis.

4. The critical period is used to identify the variation of required storage with reservoir yield. "Firm yield" is the minimum sustained reservoir release possible for a given reservoir size for a routing, analysis over the critical period.

5. Requirements are compared with firm yield to determine design reservoir size.

Such a procedure lacks any estimates related to statistical confidence levels associated with design. One point estimate of reservoir design is produce for each yield. This is the reason synthetic data generation techniques[3,4] are, in theory, useful. The-historic record is used to fit a stream flow generation model which preserves statistical attributes, including persistence. The model is used to generate synthetic records which are assumed to be traces that are of` equal chance of occurrence as the historic trace.

Each synthetic trace is subject to a critical period analysis to estimate design sizes and firm yields. For example, 30 synthetic traces might be analyzed. This provides a sample of 31 separate critical period analyses when the historic record is included. Statistical analysis of this sample produces confidence intervals and reliability levels for the storage-yield function to aid the engineer.

Thus, in theory, synthetic data generation aids the engineer in reservoir and design. In practice, the synthetic data generation models, as they are now applied, fail in this application because they fail to statistically reproduce, in expectation, the historic critical period.

Consider the following typical analysis:

1. Historic data are analyzed and fit to a stream flow generator.

2. A particular yield (Y) is desired and the critical period of the historic-record establishes a reservoir design volume of V_H.

3. The synthetic stream flow generator is applied and n traces are produced.

4. For the same yield (Y), each of the n traces is subject to critical period analysis and n synthetic reservoir designs are established, V_i, i = 1, 2, ... , n.

The expectations for V_H are that it is about equal the average V_i or $V_H = E(V_i)$. <u>This is rarely true in practice</u>; the typical result is that the historic design is on the order of the maximum synthetic design, or $V_{H-i} > \max V_i$.

The most recent approach is to use autoregressive moving, average representations 1,5,11,14 and their variants which

in the extreme reduce to the Markov model; this recent approach generates multiparameter persistence models and is not considered in this study although the results herein can be generalized to this case. The overall philosophy can be applied to autoregression, including Markov and self similar models.

The implication is that the critical period of the historic record is a more severe hydrologic event than can be statistically reproduced with synthetic data generators. Normally,' severity is defined 'for drought conditions and the critical period can be thought of as the worst drought occurring in a hydrologic record. The greater the drought the greater the storage required to buffer out its downstream effects.

This phenomenon, that the historic critical period, or drought, is of greater severity than can be captured by statistical models, has been observed by the authors for major river systems in the United States:

1. Potomac River, above Washington.
2. Patauxent River, Maryland
3. Rappahahannock River, Virginia
4. Sabine River, Texas

Informal communication with other workers[13] in this field indicate this to be a widespread phenomenon.

The problem is related to how the statistical models incorporate the handling of persistence. The simplest, first and most used approach is to assume the data are Markovian; this implies a rapidly decaying (geometric) autocovariance function, a condition described as being of "short memory." A more recent, more complex approach[12] is to assume the data are self similar; this implies a slowly decaying, autocovariance function of "infinite memory." Markovian persistence modeling is a short memory model and does not provide sufficient description of persistence based upon empiric observation. Self similar persistence modeling appears to be closer to reality but is expensive to implement. With respect to these two extremes, the state-of-the-art for persistence modeling prescribes the following steps:

1. A choice of model is made. Markov (more commonly used) or self-similar (still in research phase of development). Each of these models, after a normalization step, is described by a single parameter: a correlation parameter ,p, for the Markovian model[3,4] and a Hurst parameter, H, for the self similar model.2,7,12
Dependent upon the model, the appropriate parameter defines the models' theoretic autocovariance function.

2. The persistence parameter, p, or H, is estimated
by a point estimate. The Markov parameter is usually selected as the product moment, lag one, autocorrelation coefficient. The Hurst parameter is a function of estimates of the rescaled range and the standard deviation.

3. The persistence model, driven by its point estimate parameter, is used to generate synthetic data.

There is strong indication that this parameter fitting can be improved by a shift in fitting strategy. The stateof-the-art strategy uses point estimates. The strategy' [6]improvement suggested by the previous OWRT-Title II Catholic university project is:

1. Calculate empiric auto-correlations for 30 lags from the historic data.

2. Fit a one parameter theoretic autocovariance function to the empiric autocorrelations by method of least squares. That is, select the persistence parameter (p or H) on the basis of minimizing the sum of the squares of theoretic correlations less their appropriate empiric estimates.

This method empirically selects a Markov parameter of greater magnitude than the lag one product moment correlation for monthly data. This builds more persistence into the Markov assumption. Furthermore, Monte Carlo,studies show the least squares parameter to be robust if the underlying data are in fact Markovian.

The basic question is: does the proposed fitting strategy for persistence parameters. improve the ability of resultant synthetic data to statistically reproduce critical periods? This is the thrust of this research.

Operational Definition of Drought

A definition of drought is required in order to compare synthetic data with historic data. The analysis will evaluate the degree to which the expected drought, as, given by the synthetic data, agrees with the drought inherent in the historic data. It is understood that the historic data gave rise to the parameters- used- for synthetic data generation.

The definition will be related to the "critical period" for various lengths of drought. The operational definition is a curve or function of minimum average flow versus length of critical period based upon monthly flow data.

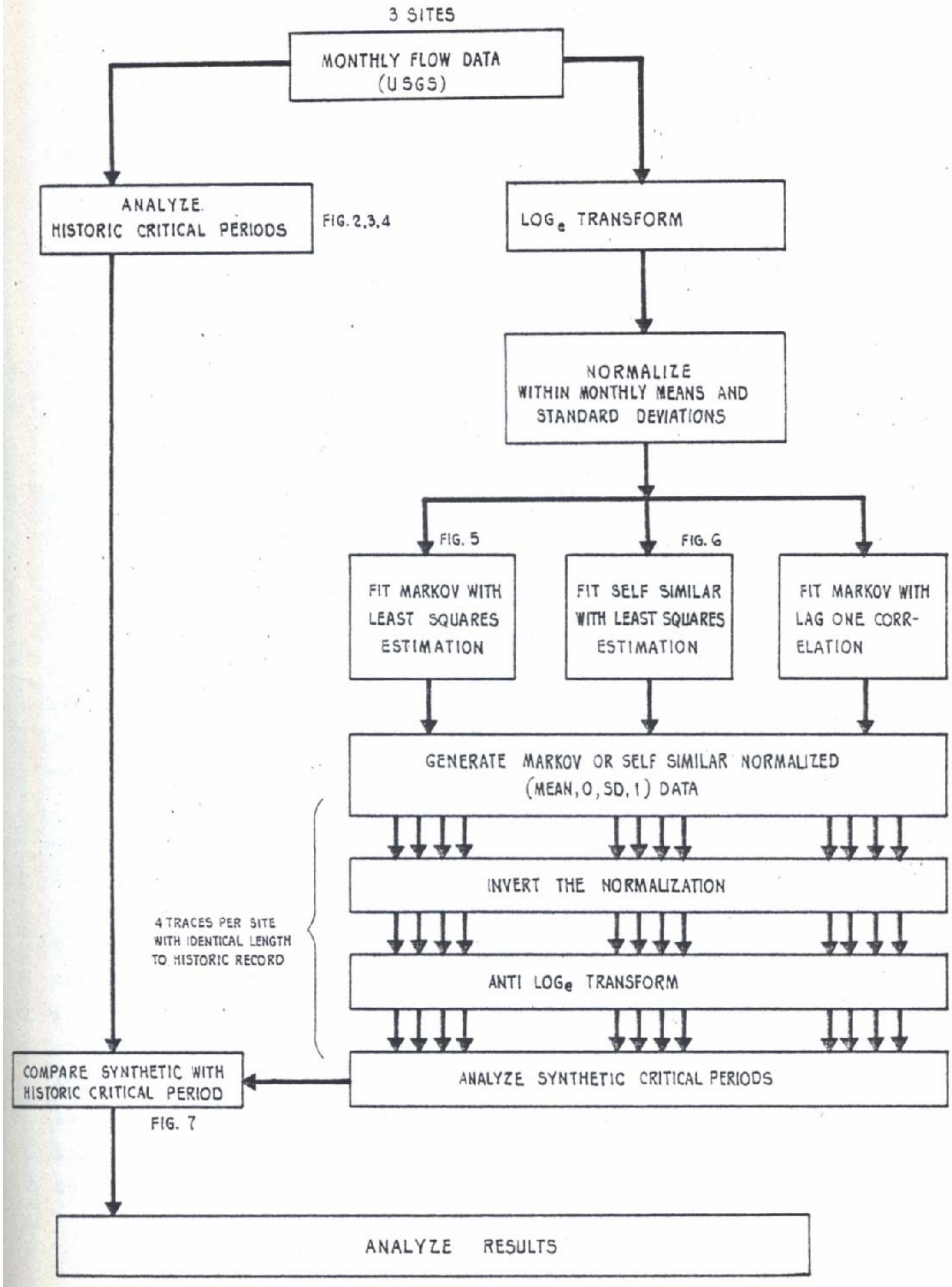
The curve is constructed as follows:

1. The record is searched for the minimum flow month. This is the average flow for a critical period of one month.
2. The record is searched for two successive months whose average flow is minimum over the set of all successive pairs of months. This is the average flow for a critical period of two months.
3. The record is searched for three successive months whose average flow is minimum over the set of all successive' triplets of months. This is the average flow for a critical period of three months.
4. And so on up to a critical period (for this study) of up to 60 months (5 years).

This definition of drought is meant to be applied to synthetic traces having exactly the same length as the associated historic record. For example, in this analysis 948 months of historic Potomac River data are analyzed. The synthetic traces that are generated all have a length of 948 months. The criteria to judge the generation mechanisms is that their average critical period curves for synthetic traces should agree with the historic critical period., Thus, this drought definition involves processing each flow trace to search for a series of minima. Note, that as the critical period increases the associated flow average approaches the average flow of the entire trace.

Methods

Figure 1 shows the overall methodology used in the analysis. The analysis of historic and synthetic critical periods follows the definition given in the previous section.



ANALYSIS FLOW

The generation and analysis associated with synthetic generation follows the logic of previous work, [17] but is briefly discussed next.

A key assumption in modeling monthly hydrologic persistence by using least squares fitted correlograms is that normalized monthly data are stationary. The monthly data correlogram shows a 12-period cycle. The usual approach for coping with this cycle in a Markov monthly stream flow Synthesis model is to compute and incorporate into the model 12 lag 1 correlations (or jump correlations of l month to the next). The approach herein is to use transforms to remove skew and periodicity.

The log transformation is a first Step and is used on the historic data to reduce skewness. Its effect on the correlogram is incidental to its purpose but the effect is noticeable.

The normalization method is periodic or cyclic. Twelve monthly means and 12 monthly standard deviations are computed, one for each month. Each log-transformed datum has its monthly mean deducted, and the difference is` divided by its monthly standard deviation; this transformation thus repeats itself
on a cycle of period of 12. The resultant residuals have mean zero and standard deviation l, and their correlogram is not period. The correlogram shows decay similar to that which would be expected for stationary Markov of self-similar data.

Although the correlation structure of normal logtransformed historic data may exhibit weak nonstationary tendencies in month to month correlations, or other subtle interactions, it is assumed to be stationary on the basis of the characteristic shape of decaying nonperiodic correlograms. Greater precision in the handling of the lag correlation structure does not appear to be warranted [8] on the basis of the economic responses of simulation models that use synthetic hydrology.

The least squares correlogram fitting method selects a smooth correlogram-function (either Markov or self-similar) by searching for that lag l correlation or Durst parameter that minimizes the square of the differences between the smooth theoretic function and the computed lag correlations.

The smooth autocorrelation function for the Markov case is

$$P_s=p^{lsl} \tag{1}$$

where s is the lag in 'months a n d ρ i s the Markov parameter.

The smooth autocorrelation funiction for the self-similar case is

$$\rho_s = 1/2(|s+1|^{2H} + |S-1|^{2H} - 2|s|^{2H}), \tag{2}$$

where s is the lag in months and H is the Hurst parameter.

The least squares fitted statistic (p is or H_{is}) of the normalized data then characterizes the autocovariance function of best fit to a sample of lag correlations by the least squares criterion. Its use in a model for generating synthetic data should produce data having a set of lag correlations; in agreement with the set of historic lag correlations equal weight is given to all lags used in the fit and information in the higher-order lags is utilized. This departs from the usual use of only the lag 1 correlation as the best estimate of a Markov model and gives weight to the higher-order lags.

Generation of residuals is followed by the inverse cyclic normalization which is-followed by inverse log transformation to obtain synthetic data.

As a further point of comparison, the Markov parameter, p, is estimated with the lag one correlation, p_1. Thus three generation schemes are driven by the three parameters, is $^{p}1s$. The Markov generation uses the Thomas-Fiering algorithm[3] . The self-generation uses Mandelbrot's fast fractional Gaussian noise method.[2,12]

The logic for the various generation steps is sequenced as shown in Fig. 1. In this work "four" traces are used to calculate the average critical period curve for each method. Four is considered to be an absolute minimum for calculation of an average and was necessitated by project budget constraints. However, for each generation scheme, each of the four traces was triggered with one of a fixed set of four starts for the pseudo random number generation.

Furthermore, each synthetic trace had the same length as the historic trace with which it is associated. For each model ($^{\rho}l_s$'.^{H} 1s, p_l) the critical period curve was generated four times. The four curves were averaged to give the "expected" critical period, curve. The expected curve *was* then compared with the historic critical period curve.

The results are presented in the next section.

<u>Results</u>

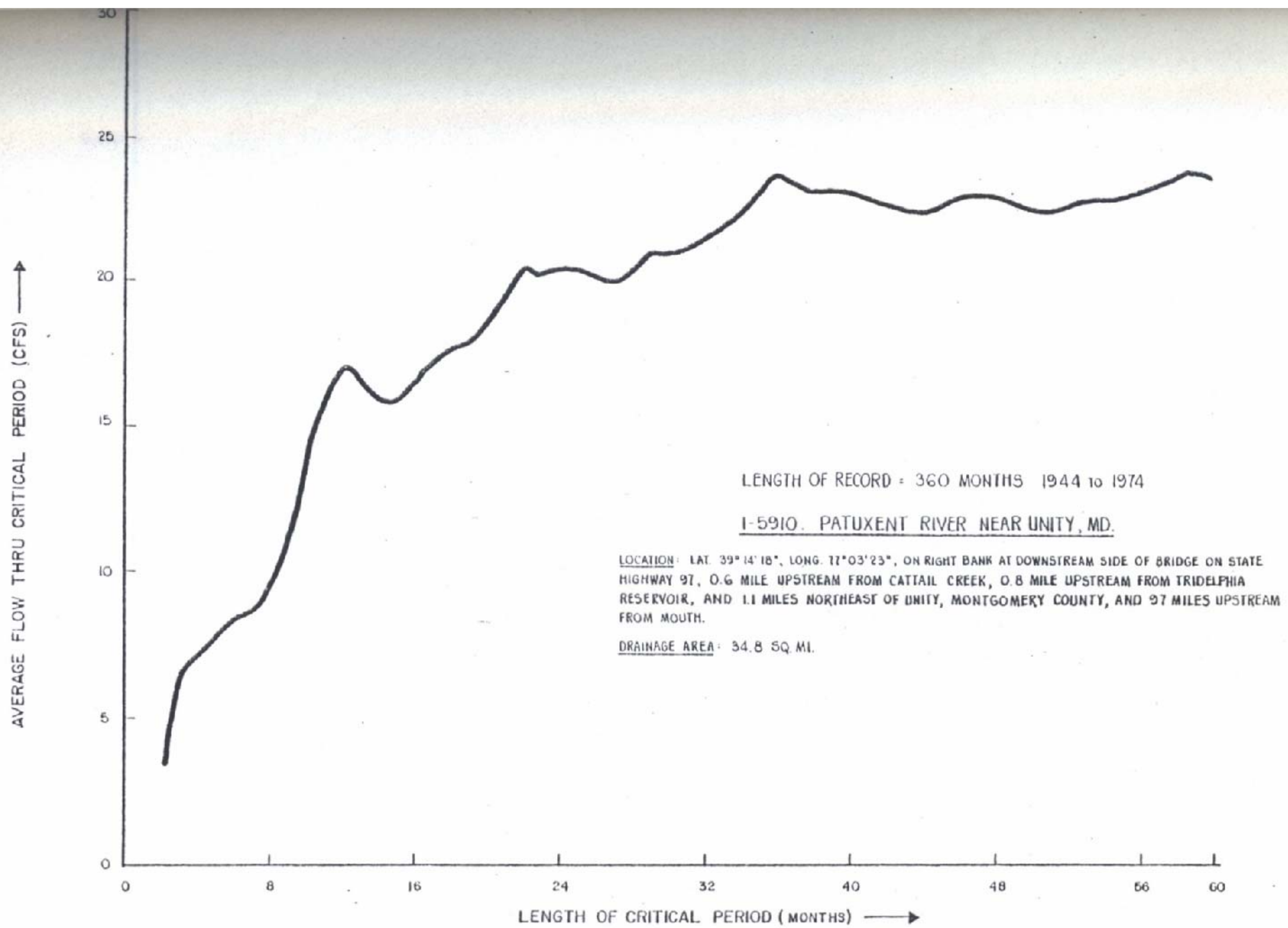
Two types of drought phenomena can be defined from the critical period analysis of historic streamflow which are shown for the three case *studies in* Figs. 2, 3 and 4.

1. Short term drought: extreme *low flaw* for *six* to eight months.

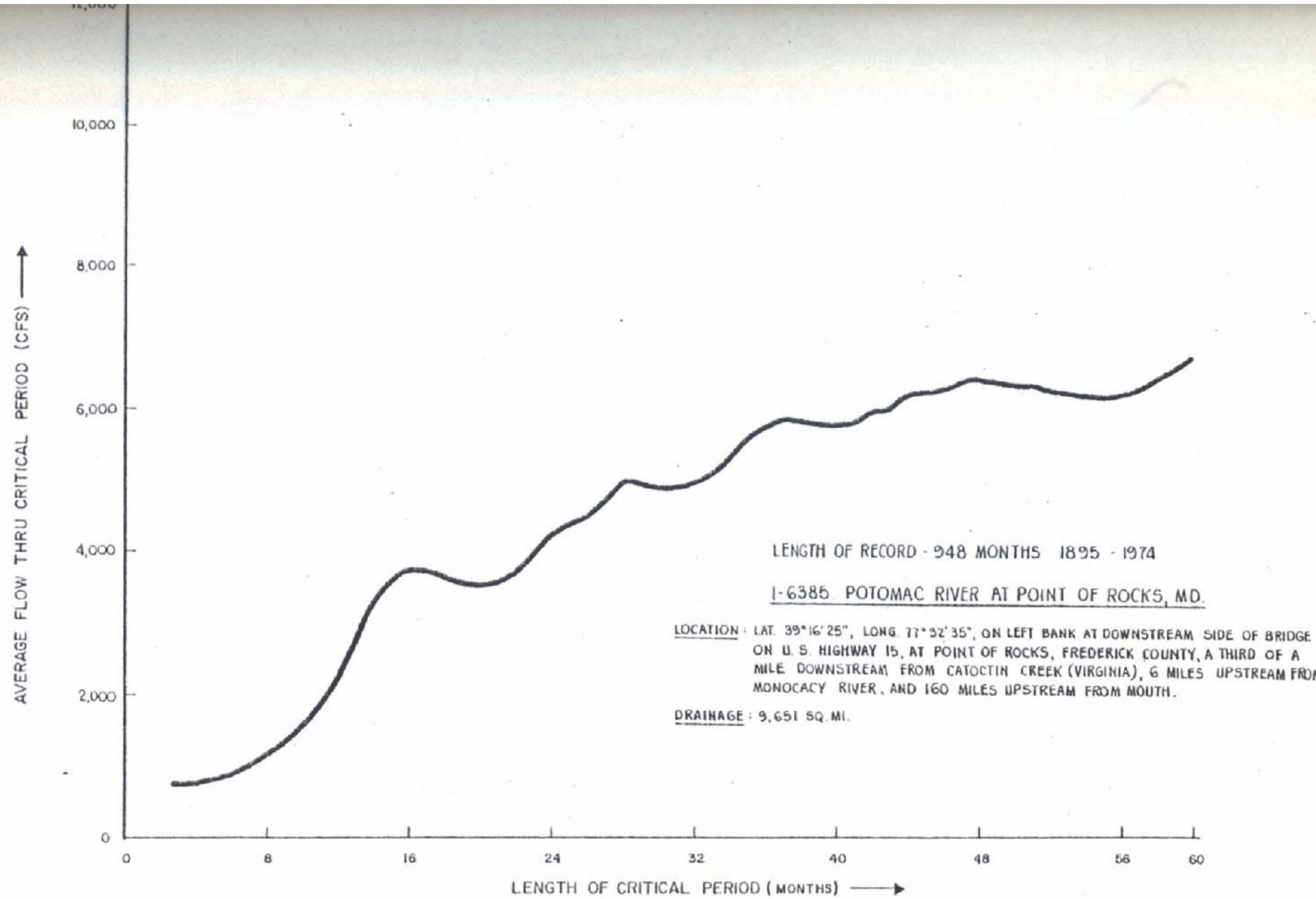
2. Long term drought: lower than historic yearly average flow for a period of three to five years (in the three study cases, approximately 2/3 of the long term annual average flow)

The first type of drought will have immediate impact on the local water resources system and crash programs will lessen the economic losses. The long-term drought will probably not be felt in its onset, but, with time, the regional water supply situation will get progressively worse and no short term alternative strategy will be available to combat the effects. Figs. 2, 3 and 4 show these effects and are used as the standard from which to measure the "goodness of fit" of synthetic data generators.

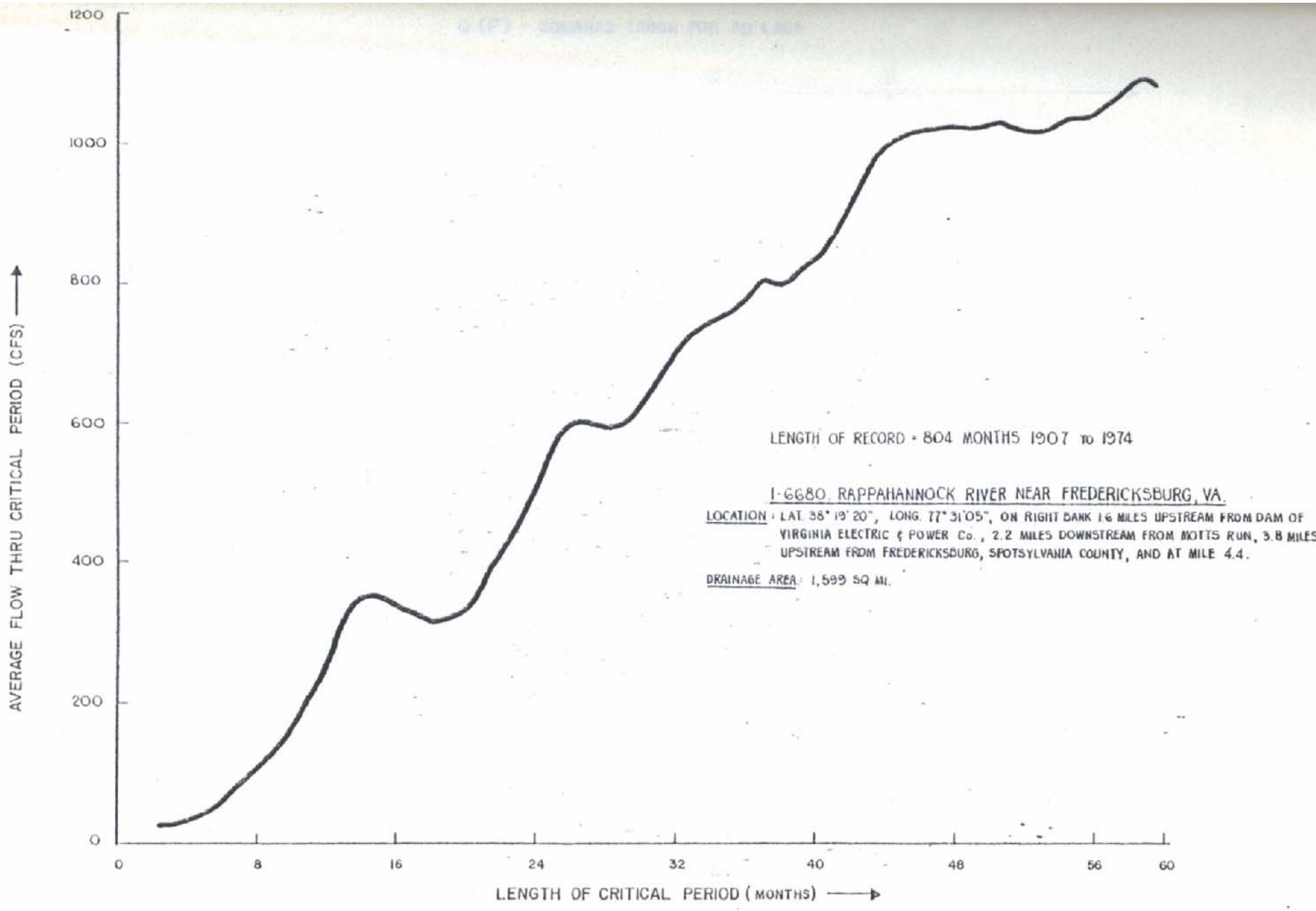
In Fig. 5 the squared errors for the theoretical autocorrelation function are' shown for the three streamflow gaging sites. The resulting minima of their squared errors follow the bias correction for the historic lag one correlation coefficient found in the literature.[8,17]



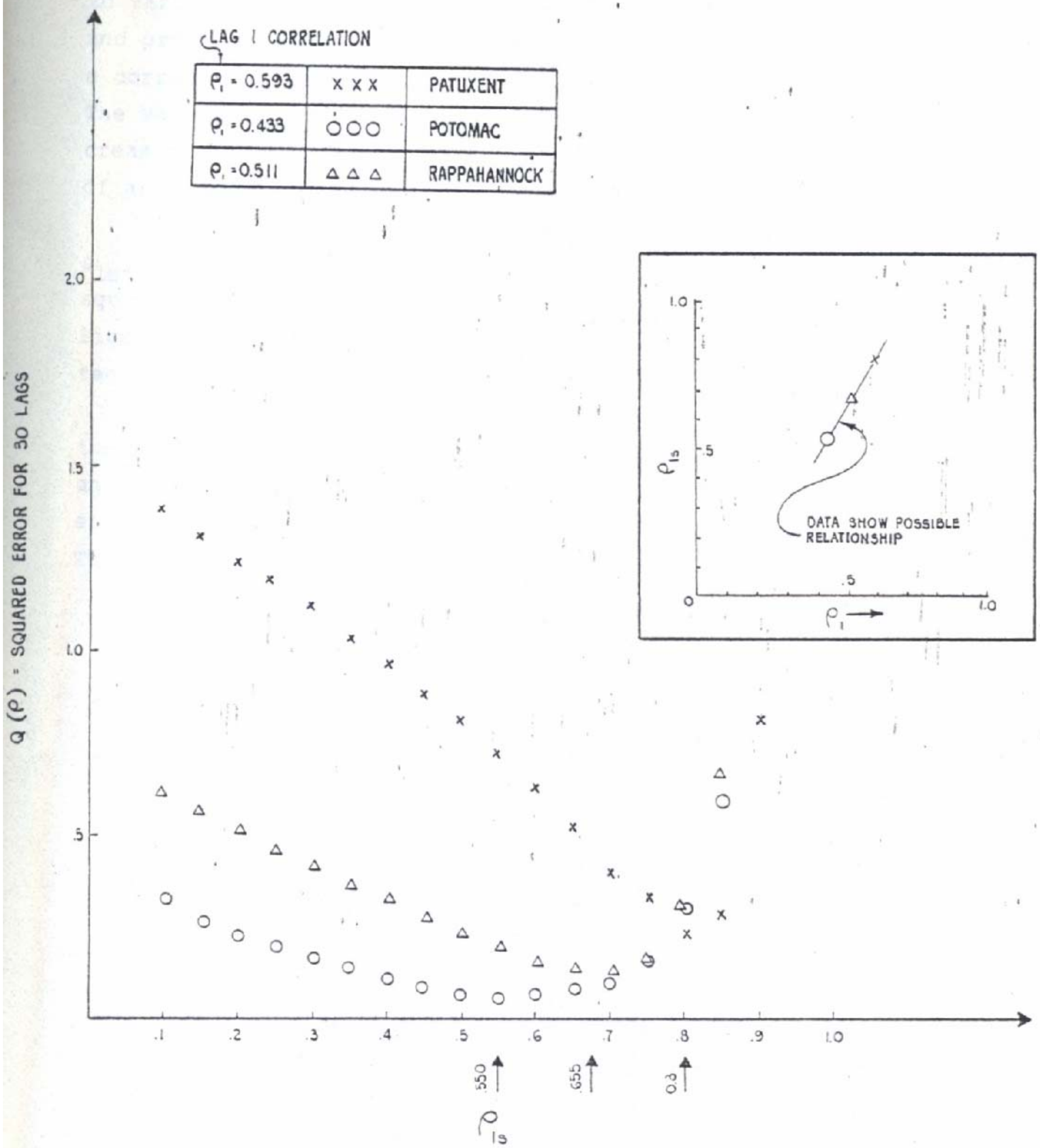
CRITICAL PATUXENT FLOWS



CRITICAL POTOMAC FLOWS



CRITICAL RAPPAHANNOCK FLOW



SQUARED ERRORS - MARKOV CASE

FIGURE 5

Fig. 6 shows the least squared errors for the self similar model parameter, H, the Hurst estimator. The behavior of the Hurst estimator as measure of long term persistence is rather erratic and *shows* extreme sensitivity to various, not *yet* fully explored conditions of physical and probably stochastic nature.[9,15] *An* apparent result is a correlation between record length and magnitude of H. The values of H tend to decrease as the record length increases, which appears to follow theoretical considerations of autoregressive processes. [3,14]

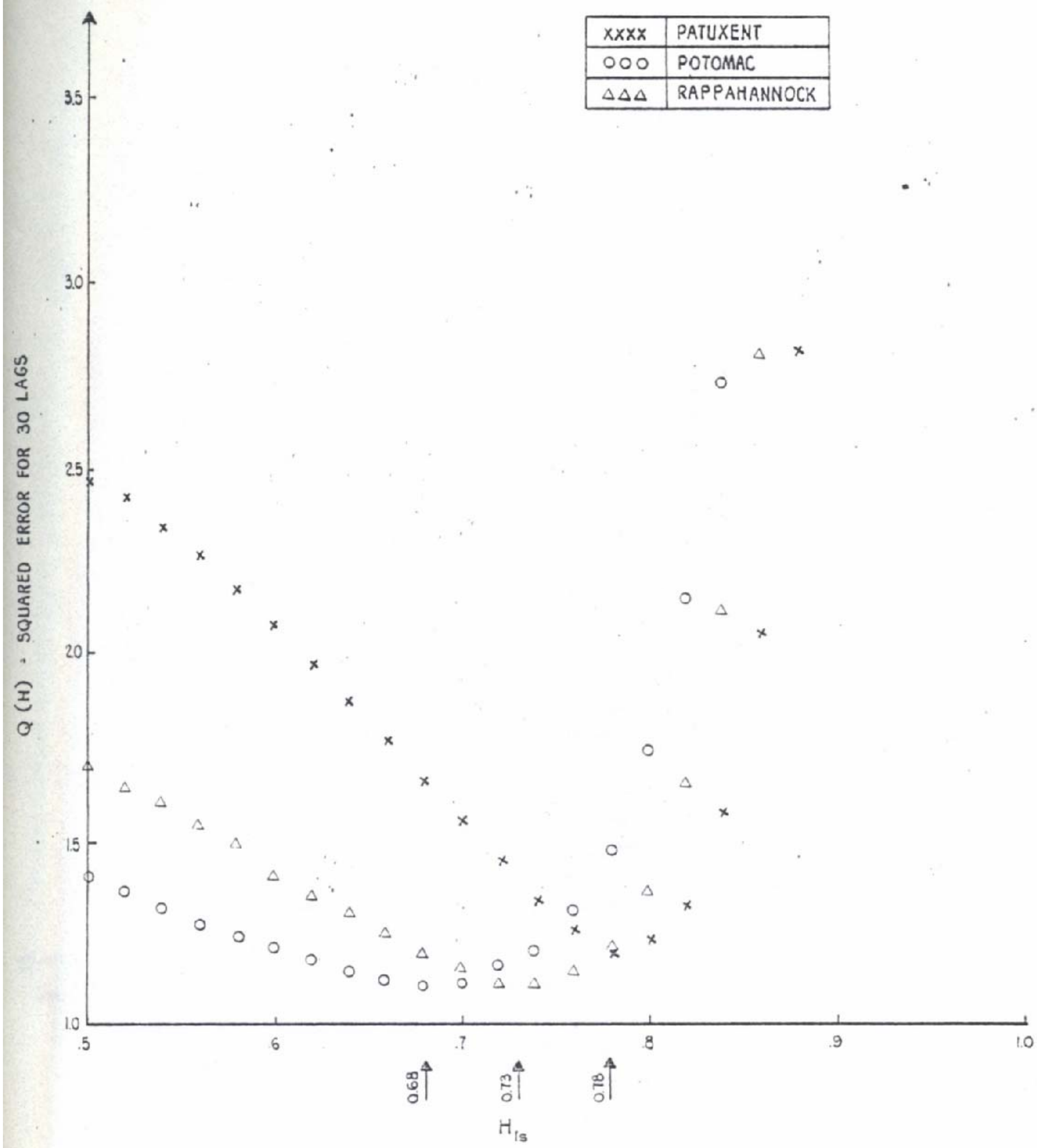
The least squares estimates of the Markov parameter, *ρ1S,* lag one estimates of the-Markov parameter, P_i, and least squares estimates of the Hurst parameter, H_{1s} , as shown in Figs.5 ahd 6 are used in simulation.

The simulation strategy is shown in Fig. 1.

The average simulated critical period is compared to the historic critical period (as given in Figs. 2, 3 and 4) and results are given in-Fig. 7. Fig. 7 shows the absolute relative errors averaged *over* the three case studies. The procedure used to generate *Fig.* 7 follows:

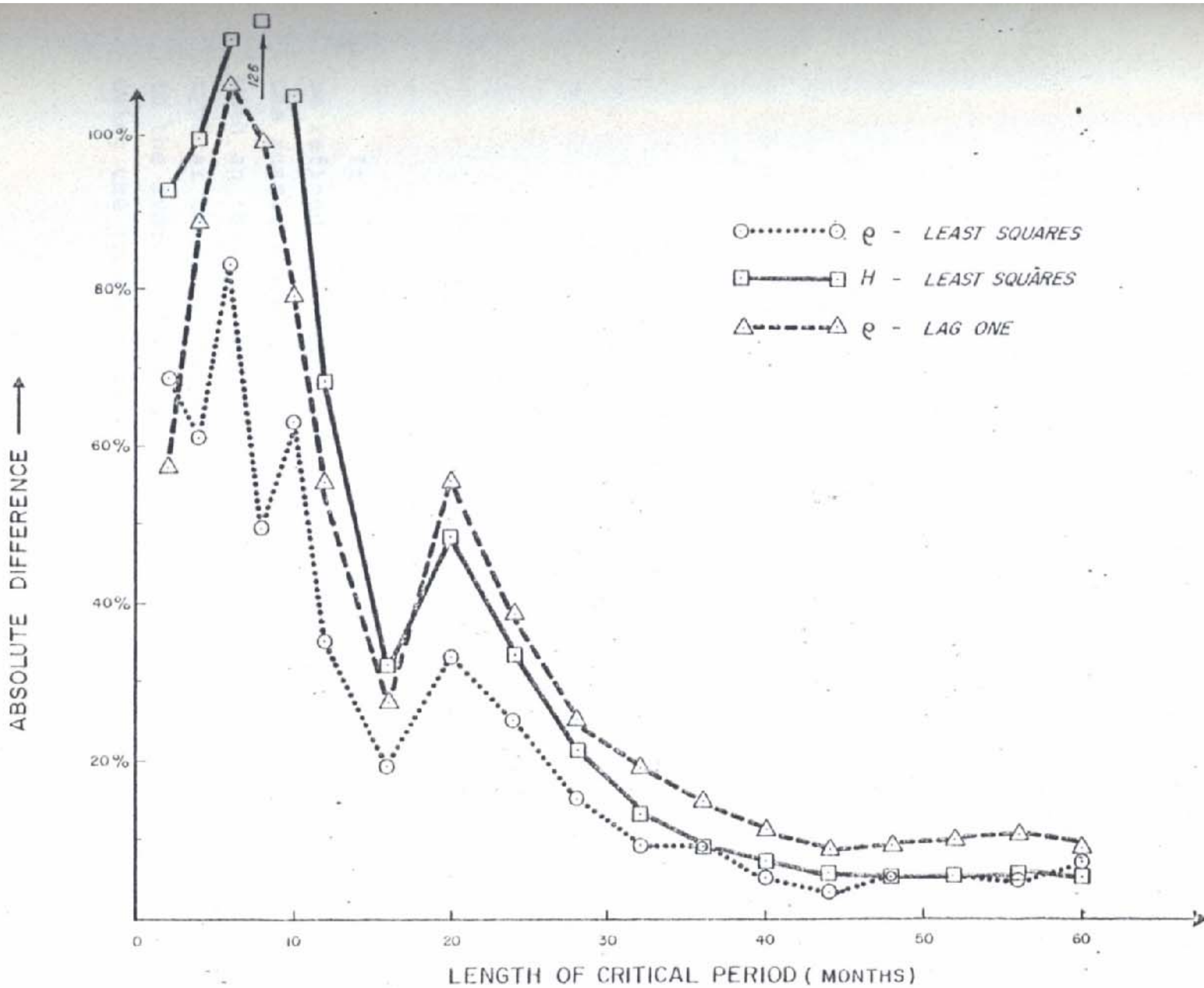
1. The errors in stream flow at *each* critical *period* length, for each synthetic sequence, are found by differencing from the historic critical period curves.
2. For each site the average of the errors is computed at each critical period, length.
3. The error is divided by the appropriate historic stream flow to obtain relative error.
4. Relative errors are averaged in absolute value to obtain the results presented in Fig. 7.

Study of Fig. 7 shows high errors for all drought estimates of critical period under three years. Over three years the errors for all *methods* are in the range of 10 percent. Over the range of critical periods studied the Markov least squares strategy produced the lowest errors (which are *still* high for extreme short term droughts), with the exception of the initial error estimate at *a 2* month critical period. The initial transposition *is* probably the result of noise from the small samples employed in *the study.*



SQUARED ERRORS SELF-SIMILAR CASE

FIGURE 6



AGREEMENT WITH HISTORICAL RECORDS

FIGURE 7

Conclusions and Recommendations

Conclusions

The conclusions are in the context of three case studies but are probably more general.

1. The least squares method, for fitting empiric to theoretic Markov correlograms produces lower errors in matching the critical periods of expected synthetic to historic flow data than other alternatives. The other alternatives are the least squares Hurst approach and the use of the lag one correlation for the Markov parameter.
2. All methods produce errors of about 10 percent or less for critical periods of over three years duration.
3. All methods produce errors of over 10 percent, in fact. From 10 percent to over 100 percent, for critical periods of under three years duration.
4. The methodology for error estimation is sound and the definition of drought is wedded to how data are used in practice to estimate firm yields.
5. The short droughts, representing the extremes of generated and synthetic data, are very error prone and additional methodological development for persistence parameter estimation is necessary for handling such extremes.

Recommendations

It is recommended that the Markov least squares method be refined to handle the short (less than 3 year) droughts. One possible approach is to generate 20 to 30 synthetic traces, with an initial persistence estimate, and screen for the typical trace producing the critical period curve nearest to the average curve of all the synthetic traces. Then, using the random number trigger of the typical curve, generate a persistence parameter that minimizes the squared differences between the resultant synthetic and historic critical period curve. This approach would probably work and many possibilities for refinement exist.

It is recommended that, until future refinements are made, that the Mlarkov least squares method be used to conduct drought simulations in a Monte Carlo context.

It is further recommended that a study of national scope be considered that focuses upon improved persistence modeling. The scope of a nation wide study on persistence should include all major rivers in the United States. Surface waters represent a large, portion of water supply for the urban centers, it is therefore of eminent importance to improve the long' term water yield prediction methods across the United States.

From this study limited to three sites around the Wash*ington* Metropolitan area it appears that the least square estimator for an autoregressive stream *flow* generator will marginally improve the long-term critical flow predictions. In other parts of the country a refined autoregressive or self similar model with a least square estimator for the Hurst coefficient might be superior. To be able to discriminate between different modeling approaches it is recommended that data for approximately four different sites in each state be analyzed.

The degree of robustness of the Markovian or self similar model assumptions will be a result of such a study. *Having* chosen the valid generating method, probability level estimates for long term water yields should improve and a case by case analysis of drought impacts will identify water management strategies to meet anticipated water shortages.

A major assumption in present hydrologic analysis is the assumption of stationarity. In general it is assumed that historic stream flow data are stationary or at most weakly non-stationary, so that the principles of time series Analysis are not violated.

An in-depth study of historic monthly stream flow data across the United States will further allow to define more precisely a discriminant criteria for the stationarity assumption of monthly stream flow data. This in turn should lead to a better understanding of “ persistence” in streamflow records. [6.9.15] Persistence is shown through the error analysis presented to this research to be an important parameter in stochastic hydrology as applied to drought. With a study of this magnitude the influence of a persistence parameter on drought modeling will be critically and exhaustively examined.

It is anticipated that a level of effort of about two to three person years over a time period of two years will resolve the various issues and lead to improved methods for drought simulations. Significant but not overwhelming computer time would be required

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ACKNOWLEDGEMENTS

This project was made possible by the Washington Technical Institute and the Catholic University of America. The OWRT grant supported the Catholic University research of the authors and contributed to the Masters Degree programs of Catholic University, students: S.K. Gyani and Anil Chaudhry.